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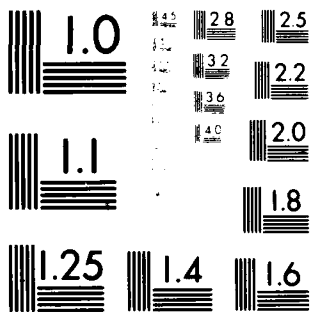

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OPTIMAL INVENTORY MODELS FOR RETAIL STOCK UNDER ALTERNATIVE DECISION RULES REGARDING RESUPPLY

by

Rosedith Sitgreaves
Sheldon E. Haber

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2 November 1981

The George Washington University
School of Engineering and Applied Science
Institute for Management Science a d Engineering

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In an earlier paper [1], a model was formulated for computing the optimal stock quantity of an item to be held by a retailer when resupply from a wholesaler is possible but uncertain. The distinguishing feature of the model is that it takes into account the cost of resupply and the probability of stock being received by the retailer in time for issue to customers. The model generalized an earlier inventory model [2] in which no resupply is possible, which in turn was a more general formalization of the classic newsboy problem (see pp. 31-32 [3]). Since the model includes as parameters the cost of transportation and the probability of receiving material on time, it can be utilized to infer which of several modes of transport is preferred for a given item.

The model described above assumed an infinite amount of stock is available for shipment to the retailer and that the wholesaler incurs no cost for holding stock. Furthermore, it assumed that the wholesaler knows if a shipment to a retailer will arrive on time for sale to a customer, and if the shipment would arrive late it is not sent at all.[1] Conversely, shipments are always made when it known they will arrive on time.

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[1]The particular context for the model is a retailer who sells customized units which are desired during a short, specific period of time and if delivery cannot be made on time the customer obtains the units elsewhere. Hence a late shipment has the same outcome as no shipment, namely, a lost sale.

In the two models to be discussed in this paper, the assumption of infinite stock and no holding cost at the wholesale level is relaxed. Instead it is assumed that W units of stock are available in the inventory system, of which T units are stocked at the retail level and the remaining (W-T) units are kept at a positive holding cost at the wholesale level. As in the earlier paper, we first develop a model in which it is assumed that the wholesaler ships requested units when it is known that the shipment will arrive on time. Under this assumption it is found that our earlier result (where an infinite amount of stock is kept at zero holding cost by the wholesaler) is a limiting case of the one considered here. We then develop a second model where it is assumed that the wholesaler always ships requested units, but only π percent of the shipments to the retailer will arrive on time.

As might be expected, in most instances the decision rule to ship requested material only if it is known that the retailer will receive it on time results in the same or lower total expected loss than the one where material in short supply is always shipped. Even so, since the marginal cost of determining if units will arrive on time is positive, the decision rule of the second model, i.e., to always ship, may be the preferred one. While the marginal cost of obtaining information is not considered in our models, were such information available it could provide a means for deciding which items are best resupplied using a push system of resupply and which items one may want to exercise more stringent transportation control so as to reduce the overall cost of operating an inventory system. As will be seen below, it appears that for most items the push system is more economical for a wide range of parameter values.

1. The Inventory System

Consider an inventory system consisting of a wholesaler and retailer and that for each item there are W units of stock (varying with each different item) of which T are placed with the retailer; the remaining (W-T) units are kept by the wholesaler. Our problem is to find the optimal value of T given W .

In this inventory system, customers are assumed to demand various quantities x from the retailer, these quantities forming the probability distribution $P(x)$ where $\sum_{x=0}^{\infty} P(x) = 1$. If $x \leq T$, the demanded units are met from shelf stocks. If x exceeds T, an order for the needed units is sent to the wholesaler.

We begin by specifying a mode of transportation for shipping units between the wholesaler and retailer, the cost of shipping units by that mode of transport, and the relative frequency with which shipments made to the retailer arrive on time. The stockout cost to the retailer for units not delivered on time and the holding costs to the retailer and wholesaler are also specified.

In particular, let

C = the unit cost of shipping additional units to the retailer by the selected mode of transportation,

Π = the relative frequency with which shipments made to the retailer by the given mode of transportation arrive on time,

D_r = the retailer's loss per unit of stock in short supply,

H_r = the retailer's cost of maintaining a unit of unused stock, and

H_w = the wholesaler's cost of maintaining a unit of unused stock.

We assume $H_w < H_r$ so that we can write

$$H_w = \alpha H_r \quad \text{where} \quad 0 \leq \alpha < 1 .$$

In calculating the optimal retail stock level T for an item, we consider two cases. First, the wholesaler ships the units in short supply only if it is known that the shipment will arrive on time, i.e., shipments are made only Π percent of the time. Second, the wholesaler always ships the units in short supply, but shipments arrive on time only Π percent of the time. These two cases are discussed below.

2. Model I: Resupply Only When Units Will Arrive on Time

For the case where T units are stocked by the retailer and $(W-T)$ units by the wholesaler, and resupply is made only when it is known that the units will arrive on time at the retailer, the total expected loss is

$$L(T/W) = H_r \sum_{x=0}^{T} (T-x)\ P(x) + \alpha H_r\ (W-T) \sum_{x=0}^{T} P(x)$$

$$+ \Pi\ (C \sum_{x=T+1}^{W} (x-T)\ P(x) + \alpha H_r \sum_{x=T+1}^{W} (W-x)\ P(x))$$

$$+ (1-\Pi)\ (D_r \sum_{x=T+1}^{W} (x-T)\ P(x) + \alpha H_r\ (W-T) \sum_{x=T+1}^{W} P(x))$$

$$+ D_r \sum_{x=W+1}^{\infty} (x-W)\ P(x)\ . \qquad (1)$$

The first and second terms in (1) are the retailer's and wholesaler's expected loss, respectively, from holding too many units of stock when the demand quantity is less than or equal to T. The third term is the expected cost of transporting additionally required units which are sent the Π percent of the time it is known units will be received on time by the retailer. The fifth term is the expected loss when units ordered by the retailer are not sent the $(1-\Pi)$ percent of the time they would arrive late (and as a result they are ordered elsewhere by customers). The fourth and sixth terms are the wholesaler's expected loss from holding too many units of stock when the demand quantity is greater than T units but less than or equal to W units. In the Π percent of the time that the wholesaler does ship additional units, he is left with the remaining $W-x$ units. In the $(1-\Pi)$ percent of the time that he does not ship, he is left with all $(W-T-1)$ units. The last term in (1) in the expected loss to the retailer when the demand quantity is in excess of W units and, therefore, this excess cannot be resupplied.

If $(T+1)$ units are stocked by the retailer and $(W-T-1)$ units by the wholesaler, the total expected loss is

$$
\begin{aligned}
L(T+1/W) = {} & H_r \sum_{x=0}^{T+1} (T+1-x)P(x) + \alpha H_r (W-T-1) \sum_{x=0}^{T+1} P(x) \\
& + \Pi\Big(C \sum_{x=T+2}^{W} (x-T-1)\,P(x) + \alpha H_r \sum_{x=T+2}^{W} (W-x)P(x)\Big) \\
& + (1-\Pi)\,\Big(D_r \sum_{x=T+2}^{W} (x-T-1)P(x) + \alpha H_r (W-T-1) \sum_{x=T+2}^{W} P(x)\Big) \\
& + D_r \sum_{x=W+1}^{\alpha} (x-W)P(x)\Big) \,. \qquad (2)
\end{aligned}
$$

Hence

$$
\begin{aligned}
L(T+1/W) - L(T/W) = {} & H_r \sum_{x=0}^{T} P(x) - \alpha H_r \sum_{x=0}^{T} P(x) + \alpha H_r (W-T-1)P(T+1) \\
& + \Pi\Big(-C \sum_{x=T+1}^{W} P(x) - \alpha H_r (W-T-1)P(T+1)\Big) \\
& + (1-\Pi)\,\Big(-D_r \sum_{x=T+1}^{W} P(x) - \alpha H_r \sum_{x=T+1}^{W} P(x) - \alpha H_r (W-T-1)P(T+1)\Big) \\
= {} & H_r(1-\alpha) \sum_{x=0}^{T} P(x) - C\Pi \sum_{x=T+1}^{W} P(x) - (1-\Pi)\,(D_r+\alpha H_r) \sum_{x=T+1}^{W} P(x) \\
= {} & H_r(1-\alpha)F(T) - C\Pi(F(W) - F(T)) - (1-\Pi)(D_r+\alpha H_r)(F(W)-F(T)) \\
= {} & (H_r(1-\alpha) + C\Pi + (1-\Pi)(D_r+\alpha H_r))F(T) \\
& - (C\Pi + (1-\Pi)\,(D_r+\alpha H_r))F(W), \qquad (3)
\end{aligned}
$$

where

$$F(T) = \sum_{x=0}^{T} P(x)$$

and

$$F(W) = \sum_{x=0}^{W} P(x) .$$

The optimal value of T is the first value of T such that

$$F(T) \geq \frac{(C\Pi+(1-\Pi)(D_r+\alpha H_r))F(W)}{H_r(1-\alpha) + C\Pi + (1-\Pi)(D_r+\alpha H_r)} . \quad (4)$$

Notice that if the amount of stock in the system is infinite and there is no cost of holding this stock at the wholesale level, $F(W) = 1$ and $\alpha = 0$ in which case the optimal value of T is the first value for which

$$F(T) \geq \frac{C\Pi + (1-\Pi)D_r}{H_r + C\Pi + (1-\Pi)D_r} ,$$

i.e., the result obtained in [1]. If in addition to $\alpha = 0$ and $F(W) = 1$, $\Pi = 0$ so that no additional units can reach the retailer on time, the optimal value of T is the first value for which

$$F(T) \geq \frac{D_r}{H_r + D_r}$$

which is the solution to the newsboy problem.

3. Model II: Resupply Regardless of Arrival Time

The second decision rule regarding resupply considered in this paper is that of always resupplying additionally requested units independent of their arrival time at the retail level. In this case we assume that while a shipment is always made, it arrives on time at the retail level only Π percent of the time.

Under these circumstances,

$$L(T/W) = H_r \sum_{x=0}^{T} (T-x)P(x) + \alpha H_r(W-T) \sum_{x=0}^{T} P(x)$$

$$+ C \sum_{x=T+1}^{W} (x-T)P(x) + \alpha H_r \sum_{x=T+1}^{W} (W-x)P(x)$$

$$+ (1-\Pi)D_r \sum_{x=T+1}^{W} (x-T)P(x) + D_r \sum_{x=W+1}^{\infty} (x-W)P(x) \ . \qquad (5)$$

Comparison of (1) and (5) indicates that the third, fourth, and fifth terms of the former are omitted and two new terms are added, namely, $C \sum_{x=T+1}^{W}(x-T)P(x)$ which is the expected cost of shipping additionally requested units to the retailer and $\alpha H_r \sum_{x=T+1}^{W} (W-x)P(x)$ which is the expected loss associated with keeping the remaining $W-x$ units.

If $(T+1)$ units are stocked by the retailer and $(W-T-1)$ units by the wholesaler, the total expected loss is

$$L(T+1/W) = H_r \sum_{x=0}^{T+1} (T+1-x)\ P(x) + \alpha H_r(W-T-1) \sum_{x=0}^{T+1} P(x)$$

$$+ C \sum_{x=T+2}^{W}(x-T-1) + \alpha H_r \sum_{x=T+2}^{W} (W-x)P(x)$$

$$+ (1-\Pi)\ D_r \sum_{x=T+2}^{W} (x-T-1)\ P(x) + D_r \sum_{x=W+1}^{\infty} (x-W)\ P(x) \ . \qquad (6)$$

Therefore

$$L(T+1/W) - L(T/W) = H_r \sum_{x=0}^{T} P(x) - \alpha H_r \sum_{x=0}^{T} P(x) + \alpha H_r (W-T-1)P(T+1)$$

$$- C \sum_{x=T+1}^{W} P(x) - \alpha H_r (W-T-1)P(T+1) - (1-\Pi)D_r \sum_{T=x+1}^{W} P(x)$$

$$= H_r(1-\alpha)F(T) - (C+(1-\Pi)D_r)(F(W) - F(T))$$

$$= (H_r(1-\alpha) + C + (1-\Pi)D_r)F(T) - (C+(1-\Pi)D_r)F(W) \ . \qquad (7)$$

The optimal value of T is the first value such that

$$F(T) \geq \frac{(C+(1-\Pi)D_r)F(W)}{H_r(1-\alpha) + C+(1-\Pi)D_r} \ . \qquad (8)$$

As can be seen by comparing (4) and (8), all other things being equal, when $\Pi = 1$ the optimal value of T is the same for both models, since additionally requested units will always arrive on time and are always shipped or, alternatively, they are always shipped and always arrive on time.

4. The Special Case of Insurance Items

In many instances, particularly in military inventory systems, it is desirable to stock a single unit in the system eventhough it is not likely to be demanded over a long period of time. Applying Model I to this situation, i.e., where $W = 1$,

$$L(T/W) = L(0/1) = \alpha H_r P(0) + (\Pi C+(1-\Pi)\ (D_r + \alpha H_r))\ P(1)$$

and $\quad L(T+1/W) = L(1/1) = H_r P(0)$

so that $L(1/1) - L(0/1) = (1-\alpha)\ H_r P(0) - (\Pi C + (1-\Pi)\ (D_r+\alpha H_r))\ P(1)$.

For the Poisson distribution with mean λ , $P(0) = \ell^{-\lambda}$ and $P(1) = \lambda P(0)$; thus

$$L(1/1) - L(0/1) = \ell^{-\lambda}[H_r(1-\alpha) - \lambda(\Pi C + (1-\Pi)(D_r + \alpha H_r))] .$$

When $L(1/1) - L(0/1) > 0$, the single unit in the system should be kept at the wholesale level. Conversely, $L(1/1) - L(0/1) < 0$ implies that the unit should be placed at the retail level.

Where the transportation system is reliable in the sense that material is delivered in a timely fashion, we can assume in the limiting case that $\Pi = 1$. For Model I, this yields the result that

$$L(1/1) - L(0/1) \gtrless 0 \quad \text{if} \quad \lambda \lessgtr \frac{H_r(1-\alpha)}{C} .$$

hence material would tend to be stocked at the wholesaler, everything else constant, when λ is small as is the case for insurance items. The same result is also obtained for Model II when $\Pi = 1$. It should be noted, however, that even for small values of λ there may be combinations of α, H_r , and C for which it is more economical to hold an insurance item at the retail level.

5. A Comparison of the Optimal Retail Stock Level T and the Total Expected Loss Under the Two Alternative Decision Rules Regarding Resupply

The models developed in sections 2 and 3 permit a comparison of the optimal retail stock level T and the corresponding total expected loss when orders placed with the wholesaler are filled only when it is certain that the additionally requested units will arrive on time versus always being filled. To illustrate the differences between the two models, we compute in Table 1 the optimal value of T for arbitrary values of each variable. In computing the figures in this table, we again assume for simplicity that the distribution of demanded units is Poisson with mean λ . To further simplify the table we assume that the holding cost

Table 1

Optimal Retail Stock Level T for Selected Values of the Model Parameters[a]

	Model I				Model II			
	H_r=5		H_r=50		H_r=5		H_r=50	
	Π=.10	Π=.95	Π=.10	Π=.95	Π=.10	Π=.95	Π=.10	Π=.95
	W = 1, λ = .05							
D_r = 5, C=5	0	0	0	0	0	0	0	0
D_r = 5, C=250	0	1	0	0	1	1	0	0
D_r = 100, C=5	1	0	0	0	1	0	0	0
D_r = 100, C=250	1	1	0	0	1	1	0	0
	W = 10, λ = 1.00							
D_r = 5, C=5	1	1	0	0	1	1	0	0
D_r = 5, C=250	2	4	1	2	4	4	2	2
D_r = 100, C=5	3	1	1	0	3	1	1	0
D_r = 100, C=250	3	4	1	2	4	4	2	2
	W = 1, λ = 10.00							
D_r = 5, C=5	1	1	1	1	1	1	1	1
D_r = 5, C=250	1	1	1	1	1	1	1	1
D_r = 100, C=5	1	1	1	1	1	1	1	1
D_r = 100, C=250	1	1	1	1	1	1	1	1
	W = 10, λ = 10.00							
D_r = 5, C=5	8	8	6	5	9	8	6	5
D_r = 5, C=250	10	10	8	10	10	10	10	10
D_r = 100, C=5	10	9	9	6	10	9	9	6
D_r = 100, C=250	10	10	9	10	10	10	10	10
	W = 20, λ = 10.00							
D_r = 5, C=5	10	10	7	6	11	10	7	6
D_r = 5, C=250	14	17	9	13	17	17	13	13
D_r = 100, C=5	16	11	11	7	16	11	11	7
D_r = 100, C=250	16	17	12	13	17	17	14	13

[a] α=.10 in all examples. λ is the mean of a Poisson distribution of demands. The definition of each of the other variables is given in the text.

for the wholesaler is substantially less than for the retailer as indicated by our choice of α , i.e., $\alpha = 10$. Four classes of items are considered having the combination of low (high) shortage cost and low (high) transport cost. Additionally, we consider cases where the amount of system stock W is less than, equal to, and greater than mean demand λ .

Referring to Table 1 it is seen that the optimal value of T is positively related to λ , D_r , and W but is inversely related to C and H_r . Some interaction effects should also be noted. For example, for Model I

(a) a low shortage cost item which is costly to ship and would rarely arrive on time is stocked at the wholesale level (see Row 2, Col. 1);

(b) a low shortage cost item which is costly to ship and would almost always arrive on time (were it kept by the wholesaler) may be stocked at the retail level (see Row 2, Col. 2) since almost always shipping the item increases the total expected loss;[2]

(c) a high shortage cost item which is unlikely to arrive on time may be kept at the retail level when the shipping cost is low as well as when it is high (see Rows 3 and 4, Col. 1, respectively) because absence of the item can result in loss of customers.

(d) However; if the high shortage cost item in (c) will almost always arrive on time, some additional units can be kept at the wholesaler, the number of additional units stocked there being larger, the lower the transport cost (compare Row 2, Cols. 1 and 2 with Row 3, Cols. 1 and 2).

Perhaps the most interesting feature of Table 1 relates not to the behavior of Model I or Model II considered by itself, but to the fact that with few exceptions the optimal retail stock levels are

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[2] From (a) and (b) it is seen that the optimal value of T may increase as Π increases (see pp. 33-34, [1]).

similar for both over a large range of values.[3] In an unconstrained push system costs are incurred which would be avoided if discretionary controls were utilized. Optimization reduces such costs in push systems like Model II by prepositioning some stock at the retail level. Yet in most instances the optimal value of T in Model II is the same or differs by only one or two units from its more sophisticated counterpart. There are several reasons for this. First, the optimal value of T depends in Models I and II, respectively, on the ratio

$$t_1 = \frac{C\Pi + (1-\Pi)(D_r + \alpha H_r))F(W)}{H_r(1-\alpha) + C\Pi + (1-\Pi)(D_r + \alpha H_r)} \tag{9}$$

and

$$t_2 = \frac{C + (1-\Pi)D_r)F(W)}{H_r(1-\alpha) + C + (1-\Pi)D_r} \ . \tag{10}$$

As indicated in Table 2 the values of t_1 and t_2 are similar for most parameter values. Second, since P(x) is a discrete function the same optimal T may be obtained even where t_1 and t_2 are different. Additionally where W is small, at most only a small number of units can be stocked at the tender.

That the two models yield similar results despite the very different decision rules underlying each can also be ascertained from Table 3 where the total expected loss is shown corresponding to the optimal value of T in Table 1. In particular it is noticed that the total expected loss is the same for both models when T = W . In this case substituting T for W in (1) and (5) yields

$$L(T=W/W) = H_r \sum_{x=0}^{W} (W-x)P(x) + D_r \sum_{x=W+1}^{\infty} (x-W)P(x) \ .$$

---

[3] While not shown, this was also found to be true of values of α other than .10 .

Table 2

Values of t[a] for Selected Values of the Model Parameters[b]

	Model I				Model II			
	H_r=5		H_r=50		H_r=5		H_r=50	
	Π=.10	Π=.95	Π=.10	Π=.95	Π=.10	Π=.95	Π=.10	Π=.95
				W = 1, λ = .05				
D_r = 5, C=5	0.5471	0.5269	0.1741	0.1044	0.6778	0.5378	0.1741	0.1044
D_r = 5, C=250	0.8683	0.9802	0.4299	0.8400	0.9814	0.9811	0.8487	0.8466
D_r = 100, C=5	0.9517	0.6839	0.6778	0.1816	0.9536	0.6888	0.6778	0.1816
D_r = 100, C=250	0.9613	0.9806	0.7256	0.8426	0.9857	0.9815	0.8820	0.8490
				W = 10, λ = 1.00				
D_r = 5, C=5	0.5477	0.5276	0.1743	0.1045	0.6786	0.5385	0.1743	0.1045
D_r = 5, C=250	0.8694	0.9814	0.4304	0.8410	0.9826	0.9823	0.8498	0.8476
D_r = 100, C=5	0.9529	0.6848	0.6786	0.1818	0.9548	0.6897	0.6786	0.1818
D_r = 100, C=250	0.9625	0.9818	0.7264	0.8436	0.9869	0.9827	0.8831	0.8500
				W = 1, λ = 10.00				
D_r = 5, C=5	0.0003	0.0003	0.0009	0.0005	0.0003	0.0003	0.0009	0.0005
D_r = 5, C=250	0.0004	0.0005	0.0002	0.0004	0.0005	0.0005	0.0004	0.0004
D_r = 100, C=5	0.0005	0.0003	0.0003	0.0009	0.0005	0.0003	0.0003	0.0009
D_r = 100, C=250	0.0005	0.0005	0.0004	0.0004	0.0005	0.0005	0.0004	0.0004
				W = 10, λ = 10.00				
D_r = 5, C=5	0.3194	0.3076	0.1016	0.0609	0.3956	0.3139	0.1016	0.0609
D_r = 5, C=250	0.5069	0.5722	0.2509	0.4903	0.5729	0.5727	0.4954	0.4942
D_r = 100, C=5	0.5556	0.3992	0.3956	0.1060	0.5567	0.4021	0.3956	0.1060
D_r = 100, C=250	0.5612	0.5724	0.4235	0.4919	0.5754	0.5729	0.5149	0.4956
				W = 20, λ = 10.00				
D_r = 5, C=5	0.5469	0.5267	0.1740	0.1043	0.6775	0.5376	0.1740	0.1043
D_r = 5, C=250	0.8680	0.9799	0.4297	0.8397	0.9811	0.9808	0.8484	0.8462
D_r = 100, C=5	0.9513	0.6837	0.6775	0.1815	0.9533	0.6886	0.6775	0.1815
D_r = 100, C=250	0.9610	0.9802	0.7253	0.8423	0.9854	0.9811	0.8817	0.8486

[a]For Model I, t is given by Equation (9); for Model II, it is given by Equation (10).

[b]See footnote a, Table 1.

Table 3

The Total Expected Loss for Selected Values of the Model Parameters[a]

	Model I				Model II			
	$H_r=5$		$H_r=50$		$H_r=5$		$H_r=50$	
	$\Pi=.10$	$\Pi=.95$	$\Pi=.10$	$\Pi=.95$	$\Pi=.10$	$\Pi=.95$	$\Pi=.10$	$\Pi=.95$
$W = 1, \lambda = .05$								
$D_r = 5$, C=5	0.74	0.72	5.21	5.01	0.93	0.73	5.21	5.01
$D_r = 5$, C=250	1.91	4.76	6.38	16.09	4.76	4.76	16.88	16.67
$D_r = 100$, C=5	4.88	1.06	9.40	5.35	4.88	1.07	9.40	5.35
$D_r = 100$, C=250	4.88	4.88	10.57	16.43	4.88	4.83	21.06	17.02
$W = 10, \lambda = 1.00$								
$D_r = 5$, C=5	8.16	8.00	50.00	45.75	9.65	8.09	45.95	45.53
$D_r = 5$, C=250	12.57	19.04	74.06	119.32	19.11	19.10	121.03	120.59
$D_r = 100$, C=5	15.72	9.75	96.50	46.23	15.82	9.83	96.50	46.00
$D_r = 100$, C-250	16.30	19.06	105.52	119.81	19.48	19.12	129.89	121.08
$W = 1, \lambda = 10$								
$D_r = 5$, C=5	34.46	34.46	34.46	34.46	34.46	34.46	34.46	34.46
$D_r = 5$, C=250	34.46	34.46	34.46	34.46	34.46	34.46	34.46	34.46
$D_r = 100$, C=5	689.22	689.22	689.22	689.22	689.22	689.22	689.22	689.22
$D_r = 100$, C-250	689.22	689.22	689.22	689.22	689.22	689.22	689.22	689.22
$W = 10, \lambda = 10.00$								
$D_r = 5$, C=5	11.00	10.84	29.05	23.40	11.64	10.92	29.30	23.40
$D_r = 5$, C=250	12.51	12.51	45.99	68.81	12.51	12.51	68.81	68.81
$D_r = 100$, C=5	131.37	130.53	178.95	148.75	131.37	130.56	178.95	148.77
$D_r = 100$, C-250	131.37	131.37	182.01	187.67	131.37	131.37	187.67	187.67
$W = 20, \lambda = 10.00$								
$D_r = 5$, C=5	17.36	16.84	92.21	76.42	21.03	17.12	95.14	76.42
$D_r = 5$, C=250	29.09	39.24	139.49	272.99	39.43	39.38	278.08	276.77
$D_r = 100$, C=5	36.67	21.52	210.45	96.88	36.86	21.70	210.45	97.04
$D_r = 100$, C=250	37.79	39.56	257.41	274.72	40.65	39.70	298.07	278.50

[a] See footnote a, Table 1.

Thus when retailer and system stock are synonymous, the total expected loss is the same as would be calculated in the newsboy problem and is independent of Π , C , and α . Comparing the total expected losses for each model, everything else constant, it is found that, as before, the two models yield similar results in most instances.

Based on the findings of this section, we conclude that Models I and II yield substantially the same outcomes and unless the marginal cost of obtaining information for the former is trivial, the push system embodied in the latter is preferable. This may account, in part,for the prevalence of such systems.

6. Summary

The two models presented in this paper provide a simple algorithm for determining the optimal quantity of an item to be stocked at the retail level given such factors as the item's shortage cost, holding costs at the retail and wholesale levels, the cost of shipping material, the probability of stock being received on time for issue to customers, and the amount of stock in the supply system. Thus the context of these models is much broader than that of the newsboy problem in which only the shortage cost and retailer's holding cost are considered.

Besides providing a vehicle for computing the optimal retail stock level when system stock is given, the models provide a basis for deciding whether to ship units in short supply to a retailer using a discretionary control, namely, whether the units will arrive on time, or to always ship units even though they may arrive late. While the inventory system modeled consists of only one retailer and one wholesaler, it nevertheless affords a means of analytically evaluating an aspect of inventory policy which otherwise could only be addressed using more complex techniques. For the very elementary inventory system examined in this paper it appears, based on the parameter values we used, that a push system is more economical than one utilizing the discretionary control of shipping shortfall units only if they will arrive on time at the retailer.

It should be recognized that in our calculation of the optimal stock quantity T at the retail level, it is assumed that W units of stock are available in the inventory system. The problem we address is how to distribute these W units between the two echelons of supply. The more complex case of how to optimally determine W and T is not examined. One way of treating this latter problem is to determine a maximum value of W , say W^* , such that the probability of demand exceeding W^* is arbitrarily small, and then to choose that combination of T and W $(0 \leq W \leq W^*)$ for which the total expected loss is a minimum. Other approaches may also be possible but are beyond the scope of this paper. Still another problem left for further research is the extension of the models to a more complex inventory system containing more than one retailer.

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